Microstrip Propagation on Magnetic Substrates—Part II: Experiment

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Abstract—Experimental data taken on microstrip built on ferrite and garnet substrates are presented and compared with theoretical values calculated from formulas derived in a previous paper which were extended to gyromagnetic media. Good agreement has been obtained between experiment and theory. In particular the observed increase in wave attenuation at frequencies near ω_m is fully explained when the frequency dependence of the characteristic impedance is taken into account.

I. Introduction

In Part I of this paper [1], we have shown how the duality principle permits one to calculate the effective permeability of a nongyromagnetic substrate once the expressions for a dielectric substrate are known. Thus it is now possible to predict the propagation characteristics of microstrip on a magnetic substrate as a function of the properties of the substrate material, and the geometrical parameters of the microstrip, against which experimental data may be compared.

In Part II we extend our design theory to gyromagnetic substrates, such as ferrites and garnets. Our analysis is supported by extensive experimental data. Particularly significant in our treatment is the fact that all losses observed on ferrite microstrip can now be accounted for and predicted. To our knowledge, this has not been done before.

Our measurements were taken for two bias conditions, namely for the substrate demagnetized and for the substrate latched, with the magnetization in the direction of propagation.

II. Permeability of Ferrite Substrates

The formulas presented in Part I are applicable, strictly speaking, only to substrate media exhibiting a scalar permeability. They must be modified to apply to gyromagnetic media, such as ferrites and garnets. For such substrates not only will the effective permeability depend on w/h, the ratio of strip conductor width to substrate height, but it also will be a function of the frequency of operation, as well as the magnitude and direction of the biasing field or magnetization.

For a TEM mode in microstrip the relative permeability of a gyromagnetic substrate biased along the direction of propagation, i.e., perpendicular to the transverse field, can be characterized by a two-dimensional tensor of the form,

$$\boldsymbol{\mu} = \begin{pmatrix} \mu & -j\kappa \\ j\kappa & \mu \end{pmatrix}. \tag{1}$$

We consider two biasing modes of operation that are of practical interest, namely, 1) the substrate demagnetized, and 2) the substrate partially magnetized. For this second case we shall be concerned only with the "latched" state, that is, the biasing state corresponding to remanent magnetization. We consider these two cases below.

A. Substrate Demagnetized

For this state, the off-diagonal tensor elements vanish, that is $\kappa=0$. Schlömann [2] has shown that for the demagnetized state the permeability tensor element $\mu=\mu_{\text{dem}}$ can be approximated by the simple expression

$$\mu_{\text{dem}} = \frac{1}{3}\left\{1 + 2\sqrt{1-\left(\frac{\omega_m}{\omega}\right)^2}\right\} \tag{2}$$

where $\omega_m=\gamma(4\pi M_s)$, and ω is the operating frequency. Here γ is the gyromagnetic ratio and $4\pi M_s$ is the saturation magnetization. This expression also has been used by Denlinger in his analysis of microstrip [3].

B. Substrate Latched

Green and co-workers [4] at this laboratory, using their extensive experimental data, have proposed the following phenomenological expressions for the diagonal and off-diagonal elements of the tensor permeability for partially magnetized ferrites,

$$\mu = \mu_{\text{dem}} + (1-\mu_{\text{dem}})\left(\frac{4\pi M}{4\pi M_s}\right)^{3/2} \tag{3}$$

$$\kappa = \frac{\gamma(4\pi M)}{\omega} \tag{4}$$

where $4\pi M$ is the magnetization. If the substrate is latched into remanence, the magnetization takes on its remanent value $4\pi M_r$.

Sandy, of this laboratory, extending his finite difference technique [5] for solving field distributions in microstrip to gyrotropic media [6], has obtained values for the inductance per unit length as a function of w/h, μ, and κ. By curve fitting to graphs of this inductance plotted against these parameters, he deduced the

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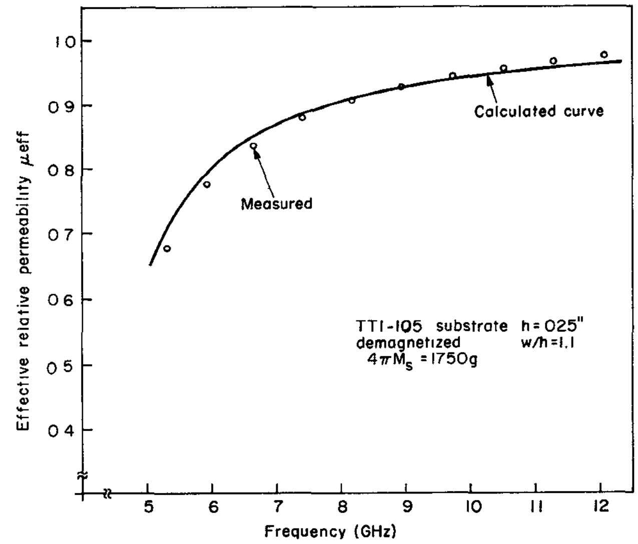

Fig. 1. Comparison of the experimental and theoretical values of the effective relative permeability for a demagnetized ferrite substrate.

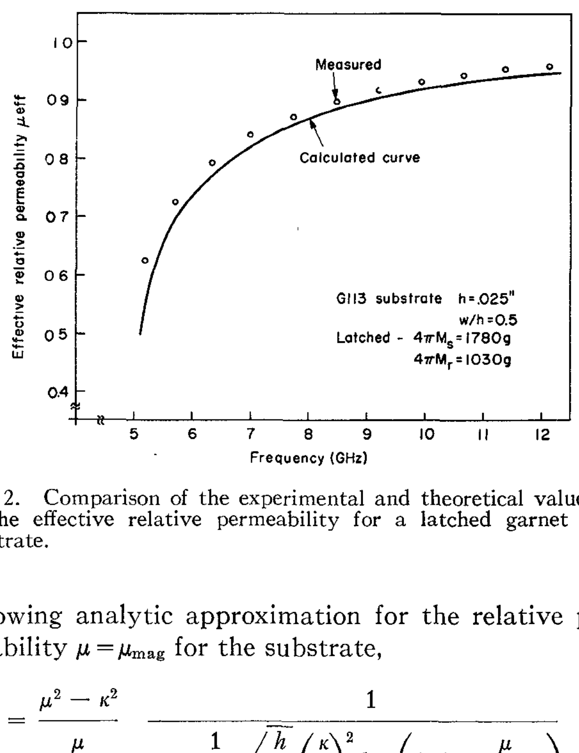

Fig. 2. Comparison of the experimental and theoretical values of the effective relative permeability for a latched garnet substrate.

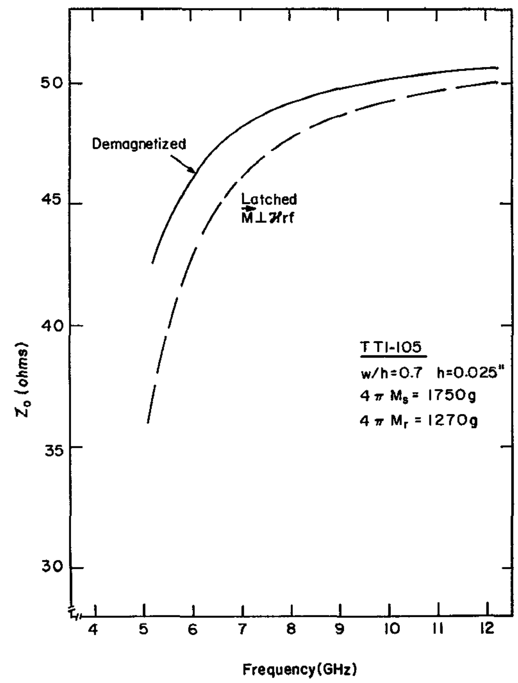

Fig. 3. Characteristic impedance of a microstrip on a ferrite substrate calculated from the measured values of relative permeability.

following analytic approximation for the relative permeability $\mu = \mu_{\text{mag}}$ for the substrate,

$$\mu_{\text{mag}} = \frac{\mu^2 - \kappa^2}{\mu} \quad \frac{1}{1 - \dfrac{1}{7}\sqrt{\dfrac{h}{w}}\left(\dfrac{\kappa}{\mu}\right)^2 \ln\left(1 + \dfrac{\mu}{\mu^2 - \kappa^2}\right)} \tag{5}$$

where μ and κ are obtained from (3) and (4) above. The denominator term containing the ratio w/h is a first-order correction arising from the gyromagnetic nature of the substrate. It is a small perturbation of about 5 percent, depending on the frequency of operation. No physical basis is claimed for (5).

The expressions for μ_{dem} and for μ_{mag} for the two states of operation now can be used to evaluate the effective permeability and the magnetic filling factors.

III. Experimental Results

The experimental data were obtained with ring resonators printed on various magnetic substrates.[1] Ring resonators, rather than open line sections, were used to reduce radiation losses to an insignificant amount [7]. From the observed resonant frequencies, the guide wavelength was determined. This provided information necessary for calculating the effective permeability. The losses were derived from the unloaded Q's of these resonance responses.

The guide wavelength corresponding to a particular resonant frequency f_0 was obtained from the formula

$$\lambda_g(f_0) = \frac{l}{n} \tag{6}$$

where l is the mean circumference of the ring and n is the mode number for that resonance, i.e., the number of wavelengths in the ring. By making l long enough to obtain many resonances in the frequency range of interest, we were able to plot a curve of λ_g versus f_0. To determine μ_{eff} from λ_g via (9) of Part I, it was first necessary for us to extract k_{eff}.

In principle k_{eff} can be calculated from Wheeler's analysis [8]. However, to do this, one must allow for the frequency dispersion observed with dielectric substrates [3], [9]. To include dispersion, we plotted against frequency the product $k_{\text{eff}}\ \mu_{\text{eff}}$ deduced from wavelength measurements. We then approximated $k_{\text{eff}}(f)$ by a linear

[1] Manufactured by Trans. Tech, Inc., Gaithersburg, Md.

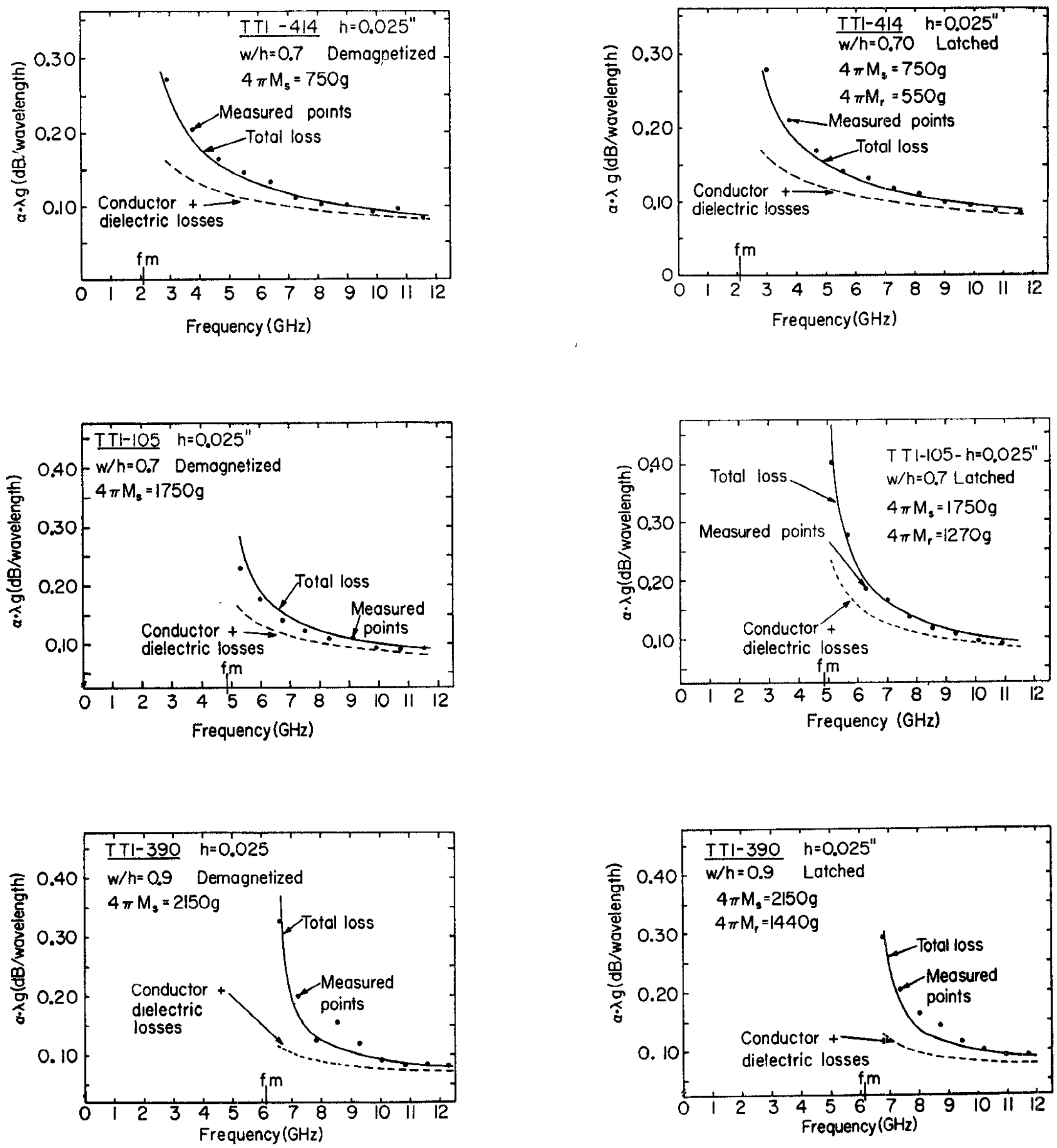

Fig. 4. Comparison of theoretical and measured loss per wavelength for three microstrips on ferrite substrates.

function of frequency having a positive slope which crossed the ordinate (zero-frequency) axis at the value of k_{eff} predicted by Wheeler's analysis. The slope of the line was adjusted to be asymptotic to the curve of k_{eff} μ_{eff} at the upper frequency end (where μ_{eff} approaches unity). This simple approximation for $k_{eff}(f)$ was satisfactory for the frequency range and microstrip dimensions considered. The effective permeability obtained by this method automatically reflects the frequency dependence of μ, as well as any possible additional dispersion, such as observed with k_{eff}. However, this additional contribution, if it did exist, was negligible in our experiments, since the deduced variation of μ_{eff} with frequency could be explained entirely by the frequency dependence of μ alone.

Figs. 1 and 2 compare measured and calculated values of the effective relative permeability for two different substrate materials and two values of w/h. Fig. 1 is for a demagnetized ferrite substrate and Fig. 2 is for a latched garnet substrate. It is evident that the agreement between theory and experiment is satisfactory.

With the values of μ_{eff} and k_{eff} obtained from these experiments, the characteristic impedance Z_0 was calculated from (8) of Part I. Fig. 3 illustrates the computed frequency dependence of Z_0 so obtained for a ferrite substrate in the demagnetized and latched states. The strong frequency variation, particularly at the low-frequency end, is a reflection of the frequency dependence of the permeability as ω approaches ω_m, characterized by (2)–(5). VSWR data, taken on a straight microstrip with good transitions, have confirmed our impedance calculations.

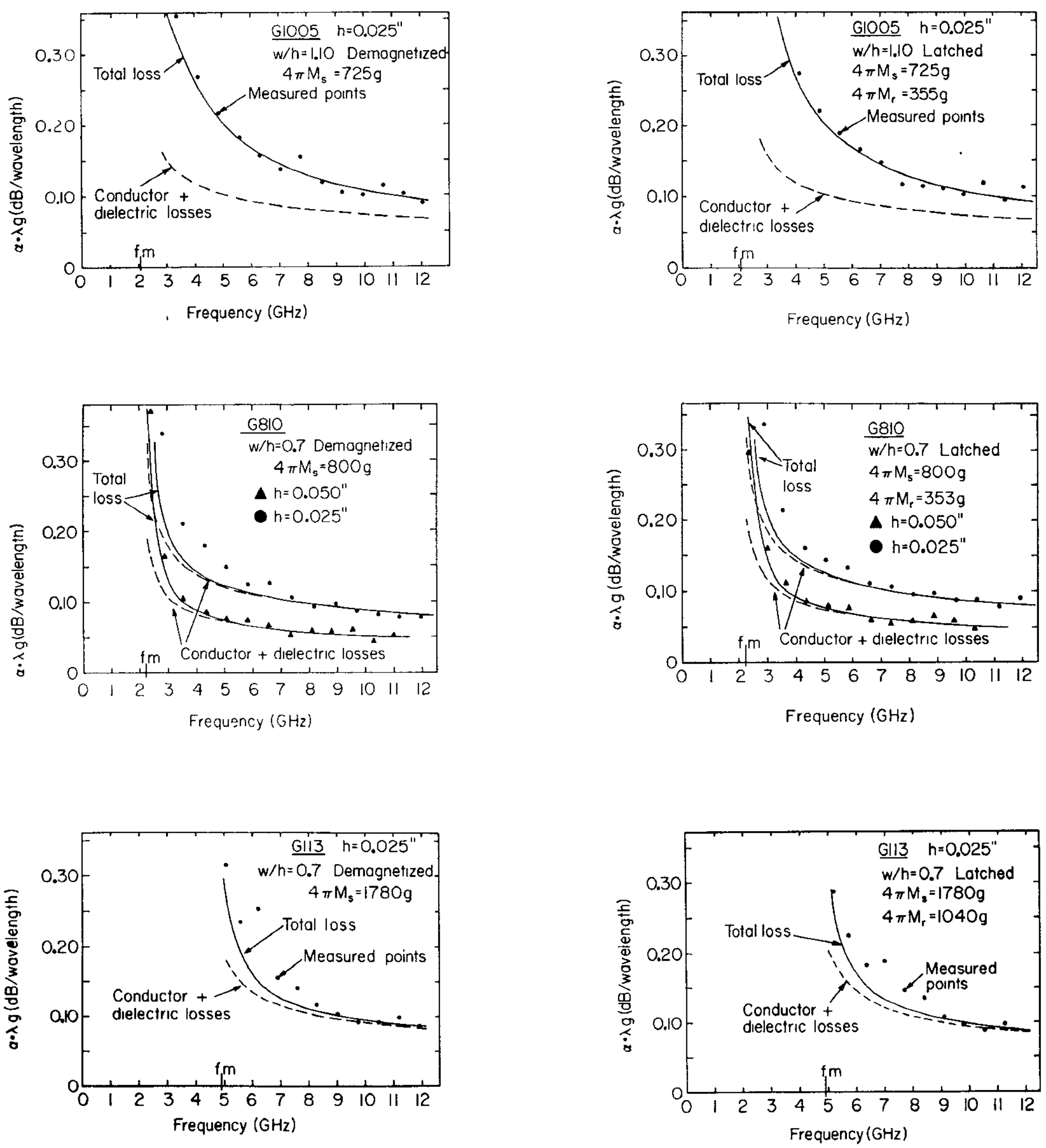

Fig. 5. Comparison of theoretical and measured loss per wavelength for three microstrips on garnet substrates.

The attenuation per wavelength was determined from measurements of the unloaded Q of the ring resonators [10]. The attenuation per wavelength, denoted here as $\alpha_T\lambda_g$, can be separated into a sum of three components, $\alpha_c\lambda_g$ due to skin losses in the strip conductor and the ground plane, $\alpha_d\lambda_g$ attributable to the dielectric losses of the substrate, and $\alpha_m\lambda_g$ arising from the magnetic losses of the substrate. The last two (substrate) components are given by (10) of Part I.

The magnetic loss tangents, $\tan \delta_m = \mu''/\mu'$, were calculated from the experimental data of Green *et al.* of this laboratory [4]. Here μ' and μ'' are the real and imaginary parts of the permeability. The dielectric loss tangents were obtained from manufacturers' data.

The conductor losses were obtained from the analysis of Pucel *et al.* [10] by correcting Z_0 and λ_g to account for the magnetic properties of the substrate in accordance with (8) and (9) of Part I. These losses were calculated for a gold metallization 5 μm thick, deposited on a substrate with a surface roughness of 20 μin rms.

We isolated the attenuation caused by magnetic losses by first calculating the attenuation attributable to the dielectric and conductor losses and comparing the sum of these two to the measured loss. We compared the difference with the computed magnetic losses and found good agreement.

Measurements were taken both on ferrite and garnet substrates,[2] for several w/h ratios and two substrate thicknesses. Both the demagnetized and the latched states were used. Figs. 4 and 5 summarize our calculated and measured results. The points represent measured data; the dashed lines, the calculated sum of conductor and dielectric attenuation; the solid line, calculated total attenuation.

[2] The substrates were annealed after machining.

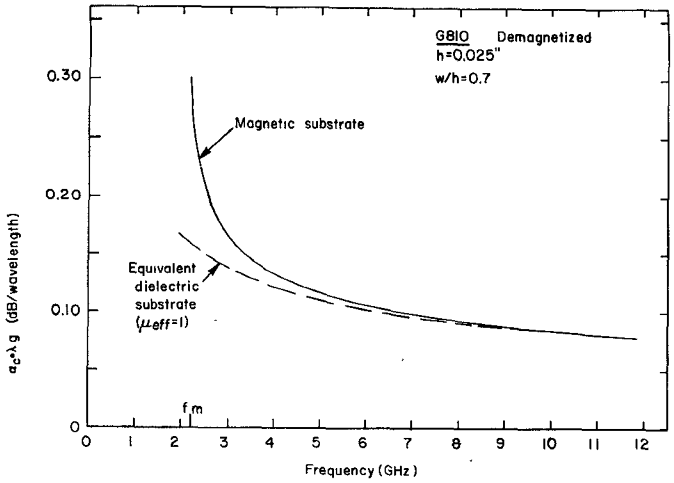

Fig. 6. Calculated conductor attenuation per wavelength for a dielectric and a garnet substrate.

The curves in Fig. 4 correspond to three ferrites with different values of saturation magnetization, hence ω_m. The agreement between the calculated and measured losses is very good in most cases. The significant increase in attenuation at the low-frequency end (as ω_m is approached) occurs for several reasons. First, magnetic losses (i.e., μ'') increase rapidly in the vicinity of ω_m. This can be seen from the difference in the solid and dashed curves. Second, and just as important, the contribution of the conductor losses increases rapidly as $\omega \rightarrow \omega_m$. This increase arises because the attenuation produced by skin losses varies inversely with the characteristic impedance Z_0 [10] and of course Z_0 drops rapidly as $\omega \rightarrow \omega_m$, as Fig. 3 shows.

The data in Fig. 5 apply to three garnets with different saturation magnetizations. Note that the data for G810 are for two different substrate thicknesses. The lower loss for the thicker substrate, of course, is attributable to the reduced copper losses, which vary inversely with thickness [10].

In general, agreement between theory and experiment is very good. This agreement shows that there are no "hidden" or anomalous sources of loss that are peculiar to magnetic substrates. Thus our accounting for the rapid increase in attenuation produced by skin losses as $\omega \rightarrow \omega_m$ removes the discrepancies between measured and calculated losses, which puzzled researchers in the past who failed to correct for the strong frequency dependence of Z_0. That the magnitude of this correction is not negligible is illustrated in Fig. 6 for one of the garnets. In this figure the conductor loss is plotted for the demagnetized garnet and for a material of identical dielectric properties but with μ_{eff} equal to 1 at all frequencies. The large increase in $\alpha_c \lambda_g$ when one approaches ω_m is shown clearly.

From the practical viewpoint the rapid rise in attenuation at low frequencies, which we now know cannot be reduced for a specific material, illustrates the importance of choosing a substrate with as small a magnetic loss (μ'') as possible, and emphasizes the necessity of operating as far from ω_m as is practical.

IV. Summary

We have proposed analytic formulas that adequately describe the microwave properties of microstrip on ferrite substrates. Good agreement is obtained between the calculated and measured properties. Our results show that wave attenuation on ferrite substrates can be predicted if proper account of the frequency dependence of the characteristic impedance is made.

Acknowledgment

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